Entangled spin-valley texture states in graphene in the quantum Hall regime

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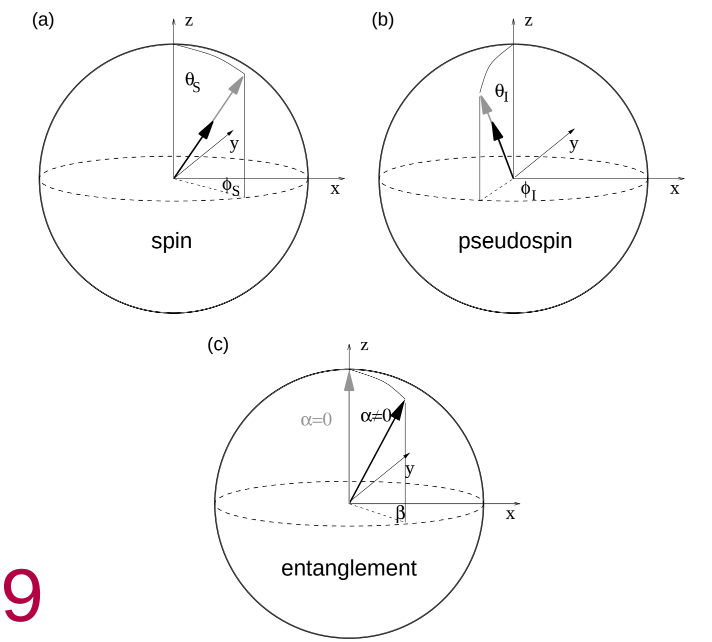
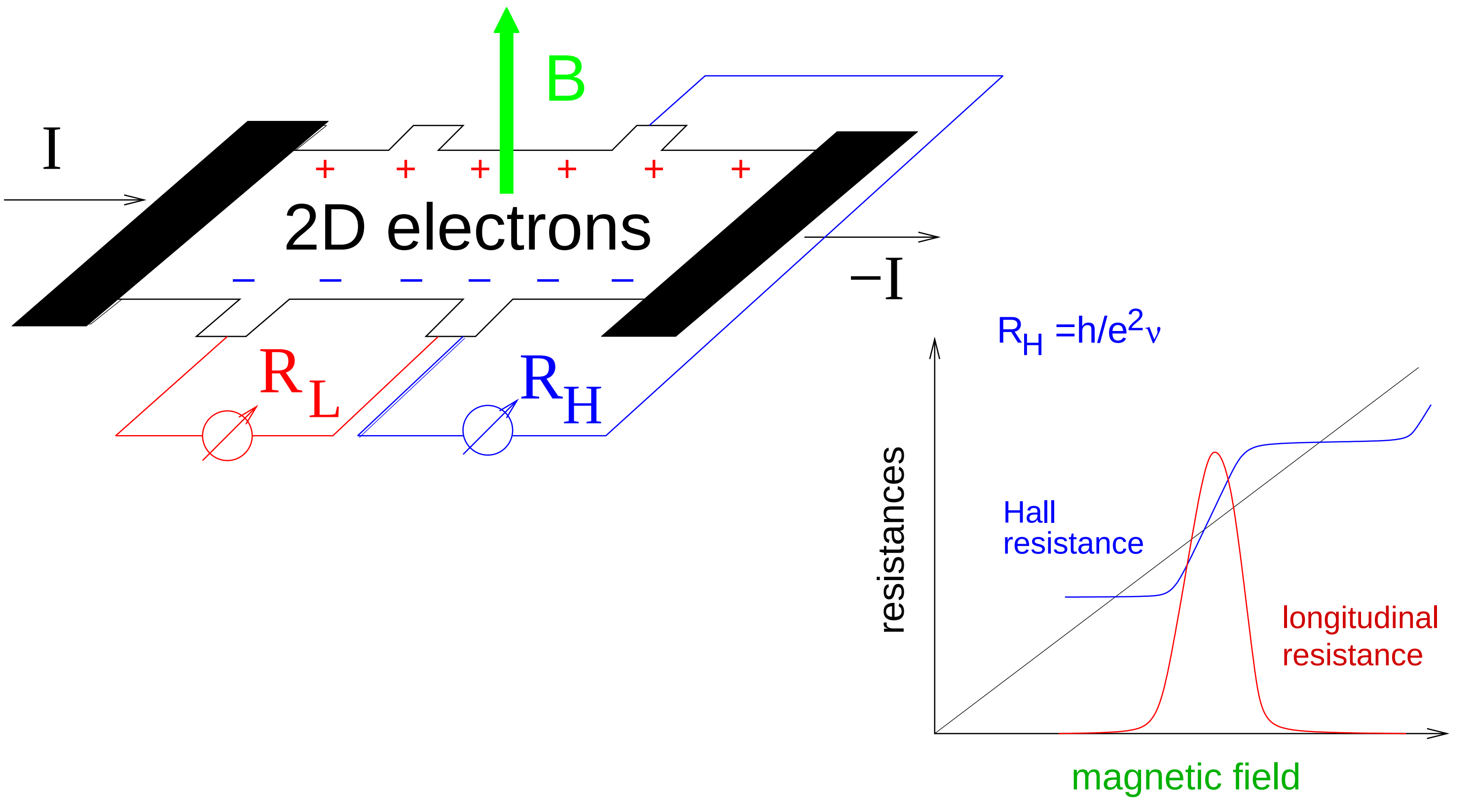


arXiv:0806.0229

Graphene Week 2008, ICTP Trieste

Quantum Hall Systems



- Quantum Hall system = 2D electrons in a perpendicular magnetic field at $T = 0$ (theoretician's limit)

Basic theoretical description

- Hamiltonian of 2D electrons (free or in a 2D crystal)

$$H(\mathbf{k}) \qquad\qquad \mathbf{k} \; : \; \text{wave vector}$$

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\+ Quantum mechanics: $[\Pi_x, \Pi_y] = -i\hbar^2/l_B^2$
with magnetic length $l_B = \sqrt{\hbar/eB}$

$\Rightarrow$ Harmonic oscillator ladder operators

$$a \propto \Pi_x - i\Pi_y \qquad a^\dagger \propto \Pi_x + i\Pi_y \qquad [a, a^\dagger] = 1$$

$\Rightarrow$ Energy (Landau) levels: $H(\mathbf{\Pi})\psi_n = H(a, a^\dagger)\psi_n = \epsilon_n\psi_n$

Degeneracy of energy levels

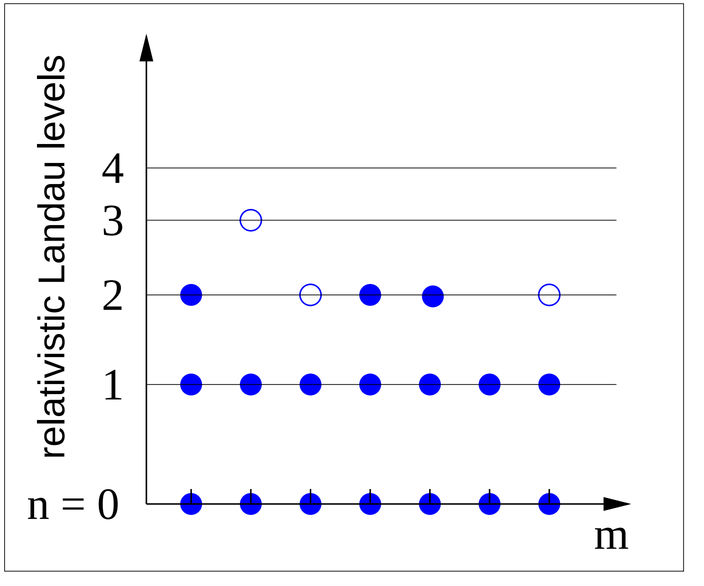
- Guiding centre operator (gauge dependent): $\tilde{\mathbf{\Pi}} = \mathbf{p} - e\mathbf{A}$
  – for symmetric gauge $\mathbf{A} = B(-y, x, 0)/2$
  $\rightarrow$ constant of motion

degeneracy: $N_B = \mathcal{A}eB/h$

filling factor: $\nu = N_{el}/N_B$

in graphene:

$\epsilon_n = \pm\hbar\frac{v_F}{l_B}\sqrt{|n|} \propto \sqrt{B|n|}$



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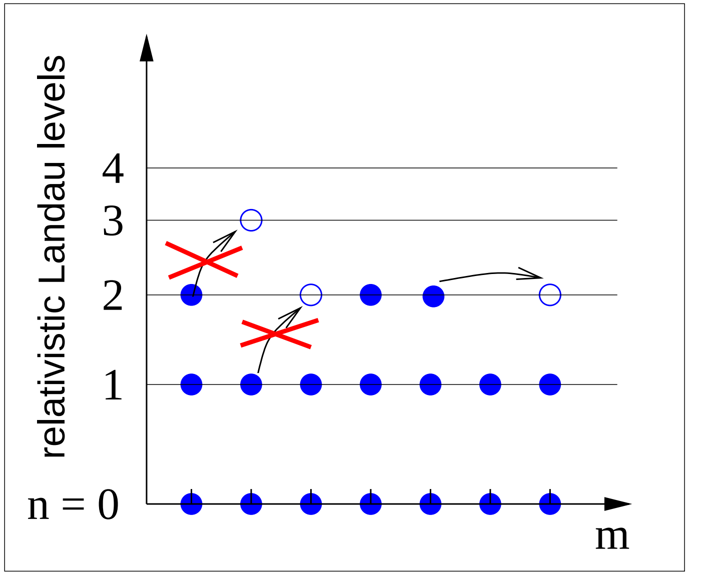


At $\nu \neq \pm 2(2n+1)$: “flat-band” limit of strong correlations

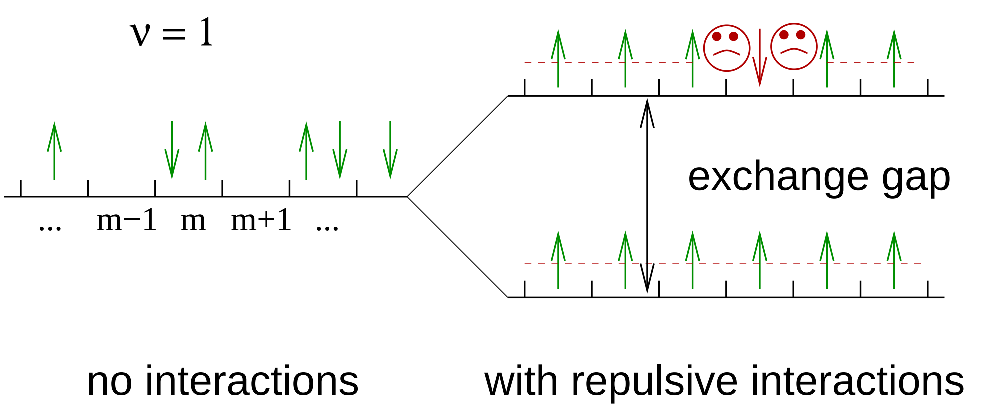
separation of energy scales, interactions lift degeneracy

Flat-band ferromagnetism at $\nu = 1$ (2DEG)

- Degenerate Landau levels have ‘quenched’ kinetic energy
- Splin-splitting due to Zeeman energy but in addition:

Coulomb repulsion favours anti-symmetric orbital wavefunction

$\longrightarrow$ spin wavefunction: symmetric (ferromagnet)



in QH systems : no kinetic−energy cost !
(LL ~ flat band)

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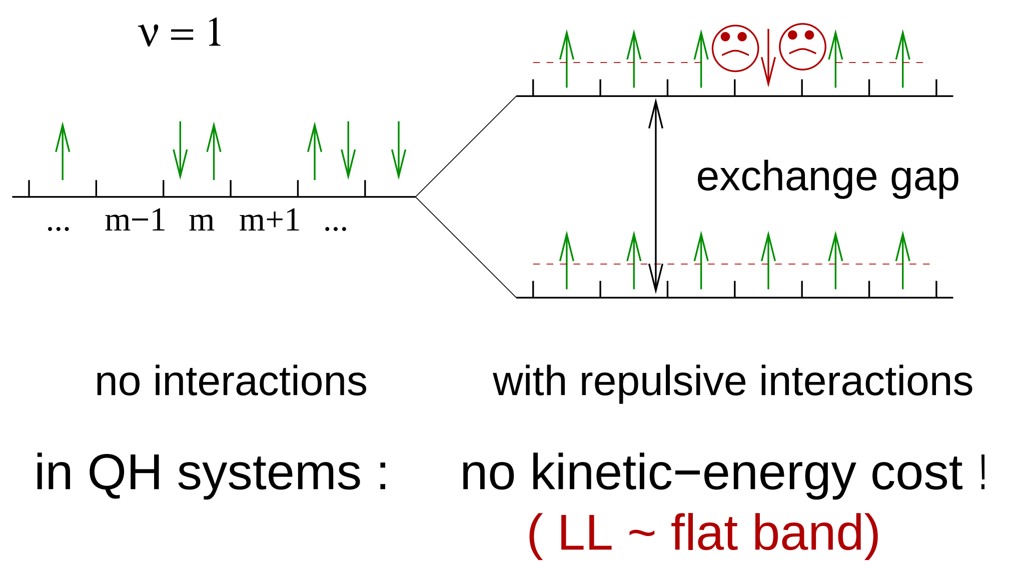
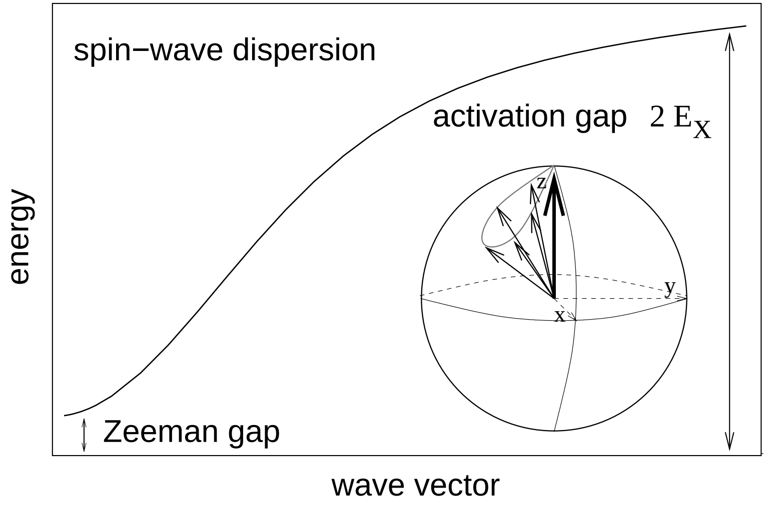


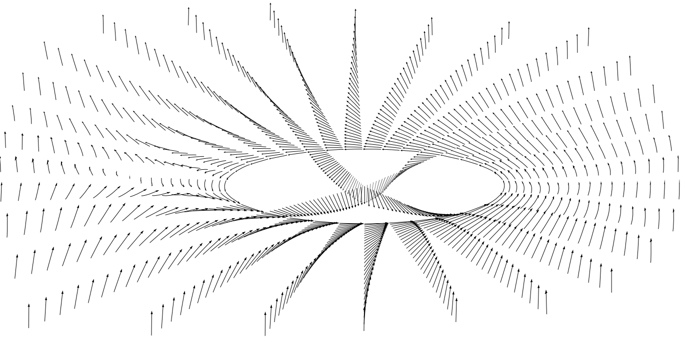
- Collective effect (‘exchange enhanced spin splitting’)
- Zeeman splitting not necessary

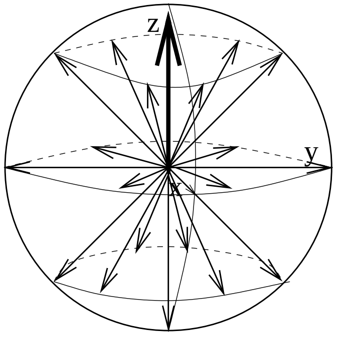
Topological spin excitations: Skyrmions

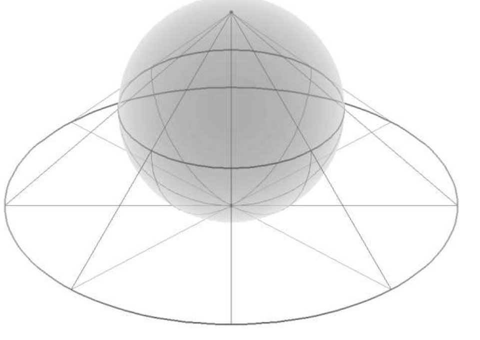
Spin waves:



Skyrmions:







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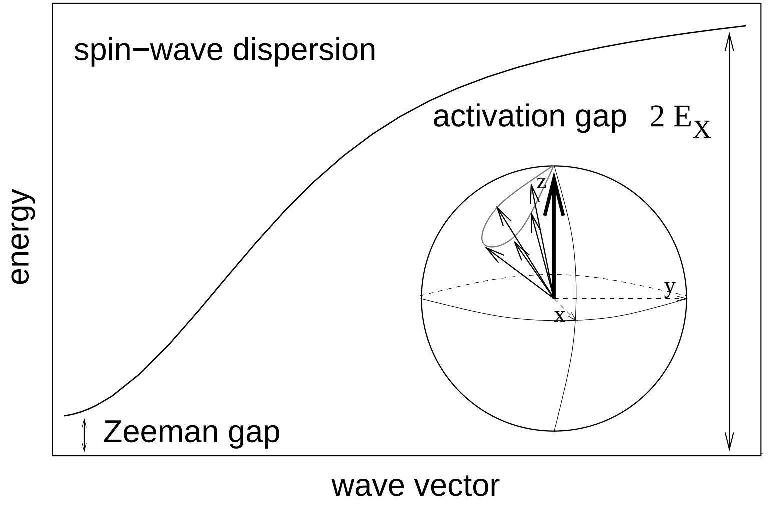
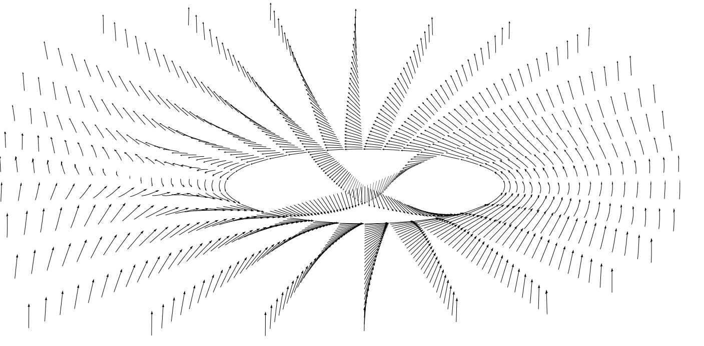
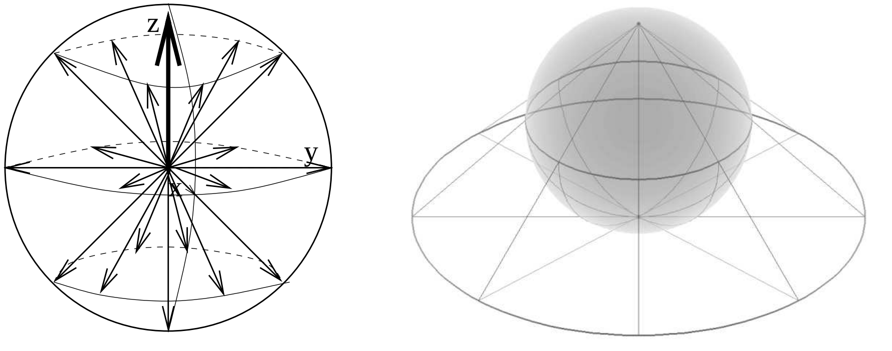


- non-topological
- charge-neutral

Skyrmions:



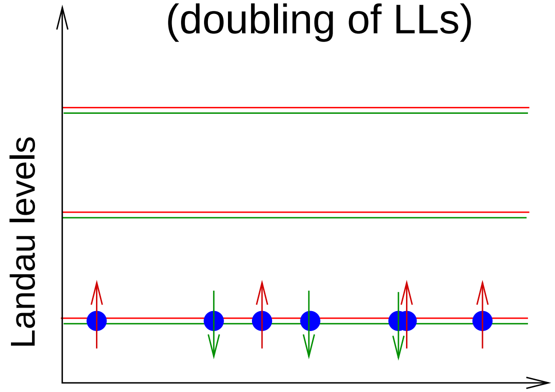


- topological
- quantised charge
- elementary excitation at $\nu = 1$ ($\rightarrow$ energetics)

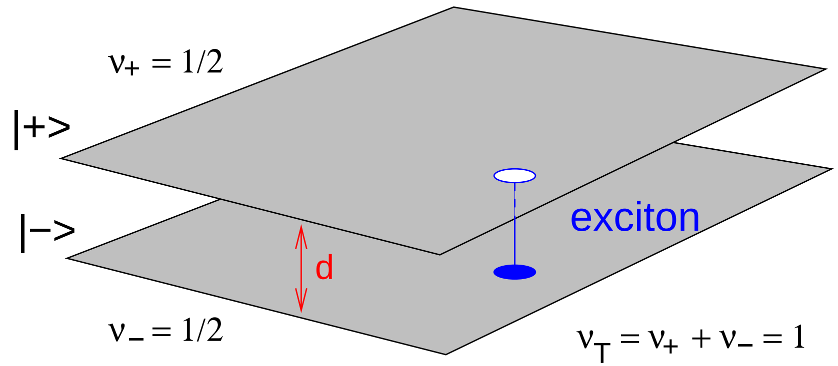
Multi-component systems (internal degrees of freedom)

Ⓐ physical spin: SU(2)

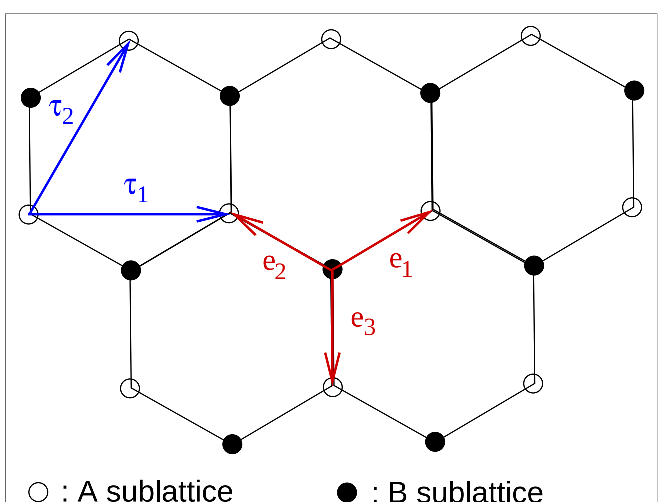
(doubling of LLs)



Ⓑ bilayer: SU(2) isospin



Ⓒ graphene (2D graphite)



two−fold valley degeneracy

→ SU(2) isospin

spin + isospin : SU(4)

Skyrmions in multicomponent systems with SU($N > 2$)

- General description of SU(N)/SU(4) Skyrmions
  Arovas, Karlhede, Lilliehöök, PRB **59**, 13147 (1999); Ezawa, PRL 82, 3512 (1999)

- SU(4) Skyrmion lattice in bilayer quantum Hall systems
  Bourassa et al., PRB **74**, 195320 (2006)

- SU(4) Skyrmions in graphene
  Yang, Das Sarma, MacDonald, PRB **74**, 075423 (2006)

Our approach:

- SU(4) description, but keeping track of the two SU(2) spin-isospin copies – spin-isospin entanglement
  Doucot, MOG, Lederer, Moessner, arXiv:0806.0229

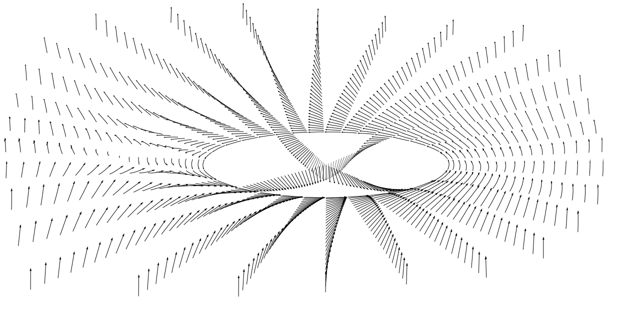
SU(2) Skyrmion parametrisation

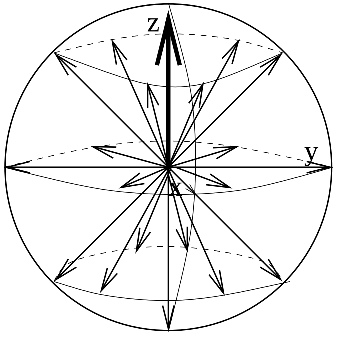
Slowly spatially varying spinor for Skyrmion of 'size' λ:

$$|\psi\rangle = \cos\frac{\theta(\mathbf{r})}{2}|\uparrow\rangle + \sin\frac{\theta(\mathbf{r})}{2}e^{i\phi(\mathbf{r})}|\downarrow\rangle = \begin{pmatrix} \cos\frac{\theta(\mathbf{r})}{2} \\ \sin\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] \end{pmatrix}$$

with 'stereographic projection'

$$\tan\frac{\theta(\mathbf{r})}{2}\exp[i\phi(\mathbf{r})] = \frac{x+iy}{\lambda} \equiv \frac{z}{\lambda}$$





Skyrmions in graphene with SU(4) symmetry

Four-component spinors

$$
\begin{aligned}
w_1 &\rightarrow \uparrow, K \\
w_2 &\rightarrow \uparrow, K' \\
w_3 &\rightarrow \downarrow, K \\
w_3 &\rightarrow \downarrow, K'
\end{aligned}
$$

For an isotropic system, w_1,...,w_4 are analytic functions of spatial coordinate z.

Topological density

- Berry connection

$$
\begin{aligned}
\mathcal{A} &= \frac{1}{i}\langle\Psi|\nabla\Psi\rangle \\
\oint \mathcal{A}\cdot d\mathbf{r} &= 2\pi Q_{\mathrm{top}}
\end{aligned}
$$

- $Q_{\mathrm{top}} = \pm 1$ for a skyrmion
- 'restores' commensurability in presence of hole (due to electric charge)

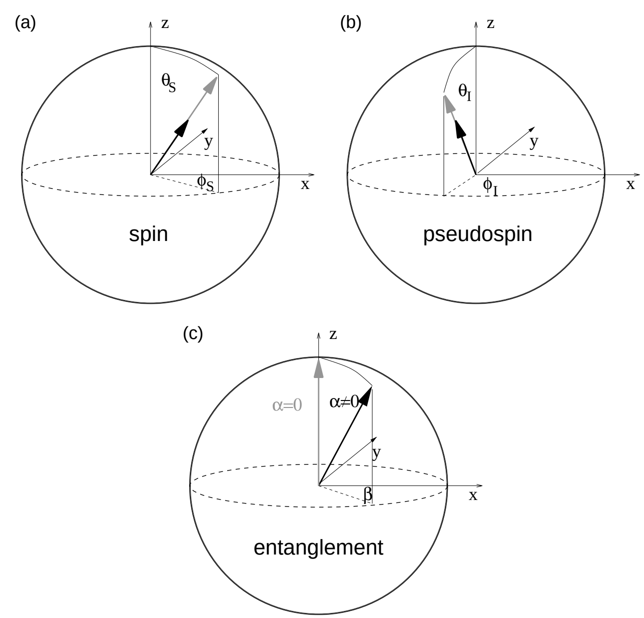
SU(2)×SU(2) parametrisation by Schmidt decomp.

Two SU(2) copies ⇒ four-spinor w (slowly varying)

- define local spinors $|\psi_{S,I}\rangle \perp |\chi_{S,I}\rangle$ for spin S and isospin I
- Schmidt decomposition for full SU(4) wavefunction:

$$\Psi(z) = \cos\left(\frac{\alpha}{2}\right)|\psi_S\rangle \otimes |\psi_I\rangle + \sin\left(\frac{\alpha}{2}\right)\exp(i\beta)|\chi_S\rangle \otimes |\chi_I\rangle$$

- S, I manifestly on equal footing
- third Bloch sphere appears
- "magnetisations" $\mathbf{m}_{S,I} = \cos\alpha\,\mathbf{n}_{S,I}$



Entanglement of spin and pseudospin

For $\sin\alpha \neq 0$, S and I are entangled:

$$\begin{aligned} \rho_S &= Tr_I\left(|\psi\rangle\langle\psi|\right) = \cos^2\frac{\alpha}{2}|\psi_S\rangle\langle\psi_S| + \sin^2\frac{\alpha}{2}|\chi_S\rangle\langle\chi_S| \\ m_S^a &= Tr\left(\rho_S S^a\right) = \cos\alpha\langle\psi_S|S^a|\psi_S\rangle = \cos\alpha\; n^a\left(\theta_S, \phi_S\right) \end{aligned}$$

Measure of entanglement:

$$\Xi \;=\; 1 - \sum_i \langle m^i_{S,I}\rangle^2 = \sin^2\alpha = 4|w_1 w_4 - w_2 w_3|^2$$

Factorisable state $\Leftrightarrow$ $\Xi = 0$

Topological density

Berry connection $\mathcal{A} = \frac{1}{i}\langle\Psi|\nabla\Psi\rangle$ and top. density $\mathcal{B} = \nabla \times \mathcal{A}$

$$\mathcal{A}(\mathbf{r}) = \sin^2\frac{\alpha}{2}\nabla\beta + \cos\alpha\left(\sin^2\frac{\theta_S}{2}\nabla\phi_S + \sin^2\frac{\theta_I}{2}\nabla\phi_I\right)$$

$$\begin{aligned}\mathcal{B}(\mathbf{r}) \quad = \quad & \cos\alpha\{\rho_{\mathrm{top}}\left[\mathbf{n}\left(\theta_S,\phi_S\right)\right] + \rho_{\mathrm{top}}\left[\mathbf{n}\left(\theta_I,\phi_I\right)\right]\} \\ & + \rho_{\mathrm{top}}\left[\mathbf{n}(\alpha,\beta)\right] \\ & + \sin^2\frac{\theta_S}{2}\rho_{\mathrm{top}}\left[\mathbf{n}(\alpha,\phi_S)\right] + \sin^2\frac{\theta_I}{2}\rho_{\mathrm{top}}\left[\mathbf{n}(\alpha,\phi_I)\right]\end{aligned}$$

where $\rho_{top} = \frac{\epsilon^{ij}}{8\pi}\mathbf{n}(\theta,\phi)\cdot\left[\partial_i\mathbf{n}(\theta,\phi)\times\partial_j\mathbf{n}(\theta,\phi)\right]$

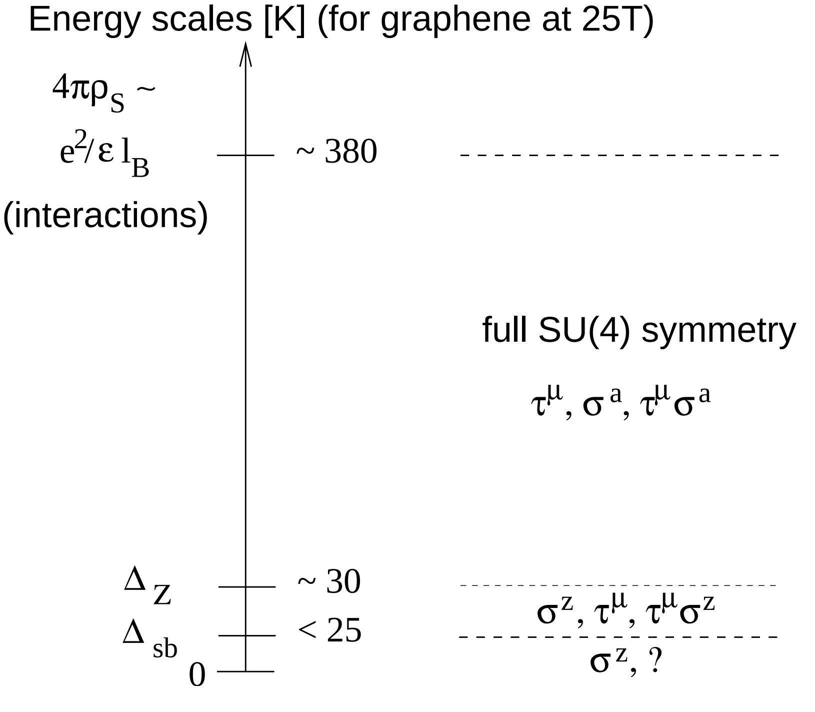
- one term depends on α,β *only*: $\rho_{top}(\alpha,\beta)$

$\Rightarrow$ corresponding Skyrmion is entangled!

Realistic anisotropies in graphene

Hamiltonian can approximately have high $SU(4)$ symmetry

- Zeeman anisotropy: $SU(2) \to U(1)$
- Graphene: valley weakly split, $O(a/l_B \simeq 0.005\sqrt{B[\mathrm{T}]})$
  MOG, Moessner, Doucot, PRB 74, 161407 (2006)



Degenerate texture families in anisotropic system

Still have U(1) generators: σ^z , τ^z and $\sigma^z \otimes \tau^z$ ($\sigma^z \otimes \tau^\mu$)

$\Rightarrow$ define entanglement operator $\mathcal{T}(\gamma) = \exp\left(i\gamma\sigma^z \otimes \tau^z\right)$

$\Rightarrow$ generates families of degenerate Skyrmions

- family members differ in entanglement $\Xi(\gamma)$

$$\Xi_{\substack{\min \\ \max}} = 4\left(|w_1 w_4| \mp |w_2 w_3|\right)^2$$

$$\begin{aligned}\Xi_{\max} &= 1 - \left[\left(|w_1| + |w_4|\right)^2 + \left(|w_2| + |w_3|\right)^2\right] \\ &\quad \times \left[\left(|w_1| - |w_4|\right)^2 + \left(|w_2| - |w_3|\right)^2\right]\end{aligned}$$

- state dependent NMR rate (spin magnetisation)!

$$\langle S^+ S^- \rangle_\gamma = \cos^2(2\gamma)\langle S^+ S^- \rangle_{\gamma=0} + \sin^2(2\gamma)\langle (S^+\tau^z)(S^-\tau^z)\rangle_{\gamma=0}$$

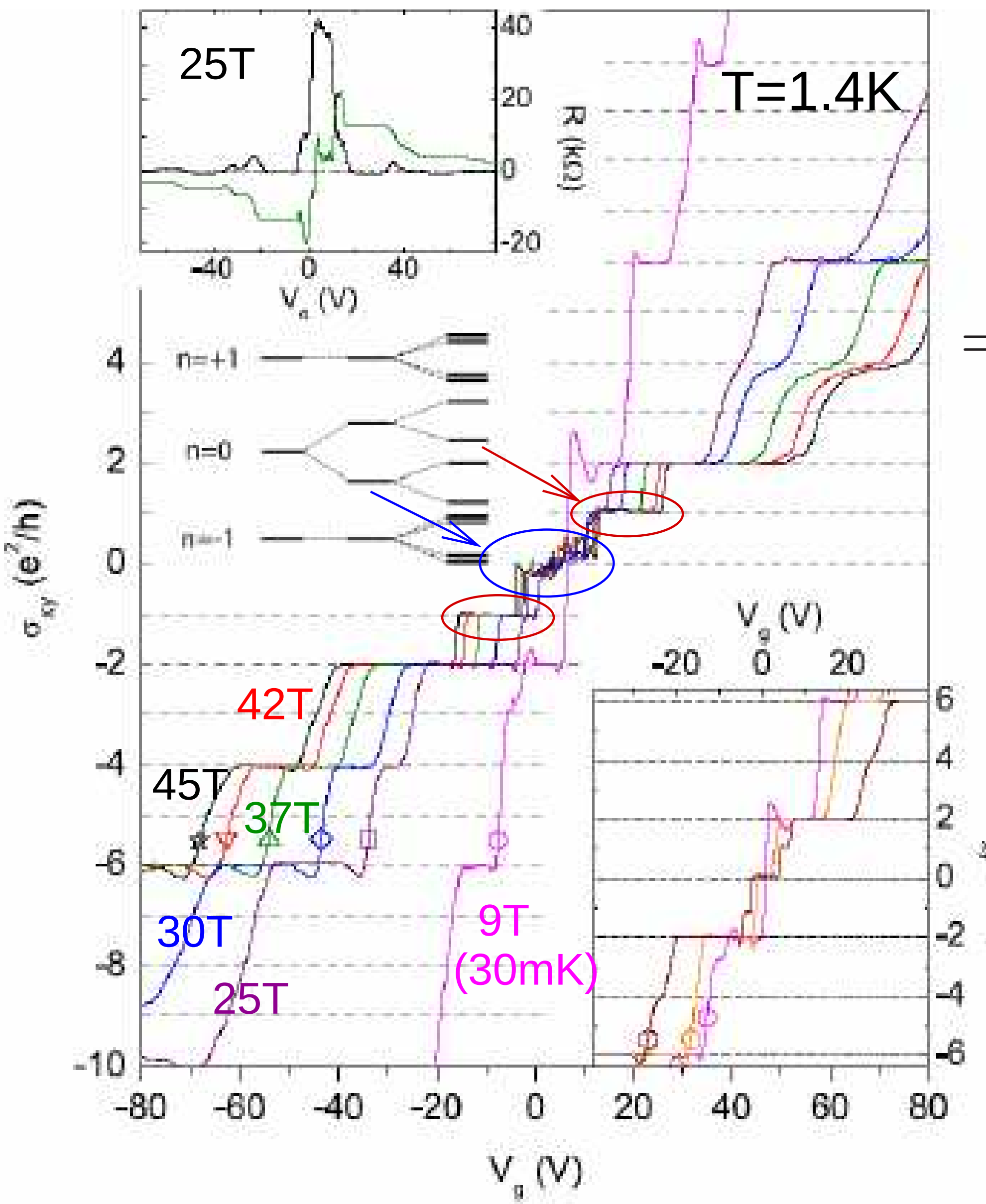
Conclusions and open questions

- Generic SU(4) textures in graphene exhibit entanglement
- Treatment of spin and isospin on equal footing
- Space-dependent entanglement as a source of topological charge
- Some symmetry operations generate entanglement
- Physical properties depend on the degree of entanglement

- Energetics of entangled textures ?
- Quantitative predictions for NMR (^{13}C in graphene) ?
- Other physical signatures of entanglement ?

$\longrightarrow$ How to probe physically the valley isospin ?

‘SU(4) Skyrmions’ in graphene?



- Plateaux at $\nu = 0, \pm 1, \pm 4$
  Zhang et al. PRL 06

$\Rightarrow$ some addt’l Landau levels individually resolved

- Simplest consideration: gaps vs. broadening Γ
- $E_{sk}^{n=0} \approx 4$meV$> \Gamma >$ $E_{sk}^{n=1} \approx 1.8$meV
- $\nu = 4$ plateau only at strong fields$\Rightarrow$ need E_Z
- other scenarios exist. . .